Pensieve header: Searching for the EK-type equation for V.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.0"];
<< betaCalculus.m
Clear[ħ]; Unprotect[C];
$PerturbativeDegree = 6;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
   sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[
   βSimplify[ω],
   βSimplify[μ]
  ];
{V, C, sol} = Get["SolutionToDegree6-120501.m"];
{V, C, sol1} = Get["SolutionToDegree6-120518.m"];
{V, C} = {
    βCollect[
      B[ω[ħ c1, ħ c2], α[ħ c1, ħ c2] t[1] h[1] +
        β[ħ c1, ħ c2] t[1] h[2] + γ[ħ c1, ħ c2] t[2] h[1] + δ[ħ c1, ħ c2] t[2] h[2]]
     ] /. {
      (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0,
      (ϵ : (α | β | γ | δ | ω | κ))^(k___)[___] :> ϵFromDigits[{k}]
     },
    βCollect[B[κ[ħ c1], 0]] /. {
      (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0,
      (ϵ : (α | β | γ | δ | ω | κ))^(k___)[___] :> ϵFromDigits[{k}]
     }
   } /. sol /. sol1
```

$$\left\{\left(1+\frac{1}{16}\,c_1\,c_2\,(1+16\,\delta_{10})\,\hbar^2+\left(\frac{1}{256}\,c_1^2\,c_2^2\left(-1-8\,\delta_{10}+128\,\delta_{10}^2+40\left(\frac{1}{12}+\delta_{10}\right)-192\,\delta_{10}\left(\frac{1}{12}+\delta_{10}\right)+19\right.\right.\right.\right.$$

```
{
 "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
 "TwistEq" → V ** Θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
 "Unitarity" → V ** (V // dA[1] // dA[2]) == B[1, 0],
 "VerticalFlipEquation" → V ** (V // dS[1] // dS[2]) == R[1, 2],
 "CapEquation" → (V ** (C // dP[12]) // dcap[1] // dcap[2]) ==
   (C * (C // dP[2]) // dcap[1] // dcap[2]),
 "SidesNonDegeneracy" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
 "CapsAndCups" → Simplify[C == (C // dS[1])]
}
```

$$\left\{\text{R4} \to \text{True},\ \text{TwistEq} \to \text{True},\ \text{Unitarity} \to \text{True},\ \text{VerticalFlipEquation} \to \text{True},\ \text{CapEquation} \to \text{True},\ \text{SidesNonDegeneracy} \to \text{True},\ \text{CapsAndCups} \to 2\,c_1\,\kappa_1\,\hbar + \frac{1}{48}\,c_1^3\,\kappa_1\left(3 + 48\,\delta_{10} + 16\,\kappa_1^2\right)\hbar^3 + \frac{1}{46\,080}\,c_1^5\,\kappa_1\left(365 - 11\,520\,\delta_{10}^2 + 7680\,\delta_{30} + 480\,\kappa_1^2 + 768\,\kappa_1^4 + 480\,\delta_{10}\left(3 + 16\,\kappa_1^2\right)\right)\hbar^5 + O[\hbar]^7 == 0\right\}$$

```
{V, C} = {V, C} /. κ1 → 0;
```

```
Simplify[C == (C // dS[1])]
```

$$O[\hbar]^7 == 0$$

```
Simplify[C == (C // dA[1])]
```

$$O[\hbar]^7 == 0$$

```
C
```

$$\left(1 + \frac{1}{32}\,c_1^2\,(1 + 16\,\delta_{10})\,\hbar^2 + \frac{c_1^4\left(-3+768\,\delta_{10}^2+160\left(\frac{1}{12}+\delta_{10}\right)-768\,\delta_{10}\left(\frac{1}{12}+\delta_{10}\right)+768\left(\frac{1}{12}+\delta_{10}\right)^2+128\,\delta_{30}+\frac{2}{3}\,(13-192\,\delta_{10}-2304\,\delta_{10}^2+576\,\delta_{30}\right.}{6144}\right.$$

```
C /. {c1 → 1, δ10 → -1 / 16, δ30 → -23/768}
```

$$\left(1 + \frac{(424+230\,400\,\gamma_{12}+122\,880\,\delta_{23}+61\,440\,\delta_{41})\,\hbar^6}{44\,236\,800} + O[\hbar]^7\right)$$

```
Solve[((14/3 + 128 δ30 + 2/3 (16 + 576 δ30)) == 0]
```

$$\left\{\left\{\delta_{30} \to -\frac{23}{768}\right\}\right\}$$

Cup equation

```
Simplify[
 (((C // dS[1] // dP[12]) ** (V // Inverse)) // hη[1] // hη[2]) ==
  (C // dS[1]) (C // dS[1] // dP[2])
]
```

True

Top cap left punctured.

```
ek1 = V * (C // dP[3]) // tη[1] // dm[2, 3, 2] // dS[2] // hm[1, 2, 2] // dP[2 → 0]
```

$$\left(1+\left(\frac{c_0^2}{32}+\frac{1}{2}c_0^2\,\delta_{10}\right)\hbar^2+\left(\frac{73\,c_0^4}{18\,432}+\frac{1}{64}c_0^4\,\delta_{10}-\frac{1}{8}c_0^4\,\delta_{10}^2+\frac{1}{12}c_0^4\,\delta_{30}\right)\hbar^4+\left(-\frac{6077\,c_0^6}{132\,710\,400}+\frac{1}{160}c_0^6\,\gamma_{12}+\frac{209\,c_0^6\,\delta_{1}}{921\,600}\right.\right.$$
$$\left.\left.t[0]\right.\right.$$

```
ek1 /. {δ10 → -1 / 16}
```

$$\left(1+\left(\frac{23\,c_0^4}{9216}+\frac{1}{12}c_0^4\,\delta_{30}\right)\hbar^4+\left(-\frac{323\,c_0^6}{8\,294\,400}+\frac{1}{192}c_0^6\,\gamma_{12}+\frac{1}{360}c_0^6\,\delta_{23}-\frac{7\,c_0^6\,\delta_{30}}{4320}+\frac{1}{720}c_0^6\,\delta_{41}\right)\hbar^6+O[\hbar]^7\right.$$
$$\left.t[0]\right.$$
$$\left.\frac{c_0\,\hbar}{48}+\left(\frac{2}{5}\right.\right.$$

No solutions to ek1==1!

Right cup left punctured.

```
ek2 = V * (C // dP[3] // dS[3]) // dm[3, 2, 2] // hη[2] // tη[1] // dm[1, 2, 0]
```

$$\left(1+\left(\frac{c_0^2}{32}+\frac{1}{2}c_0^2\,\delta_{10}\right)\hbar^2+\left(\frac{73\,c_0^4}{18\,432}+\frac{1}{64}c_0^4\,\delta_{10}-\frac{1}{8}c_0^4\,\delta_{10}^2+\frac{1}{12}c_0^4\,\delta_{30}\right)\hbar^4+\left(-\frac{6077\,c_0^6}{132\,710\,400}+\frac{1}{160}c_0^6\,\gamma_{12}+\frac{209\,c_0^6\,\delta_{1}}{921\,600}\right.\right.$$
$$\left.\left.t[0]\right.\right.$$

```
False && PerturbativeSolveAlways[ek2 == B[1, 0], ħ, 6, {c0}]
```

```
False
```

```
ek2 /. {c0 → 1, δ10 → -1 / 16}
```

$$\left(1+\left(\frac{23}{9216}+\frac{\delta_{30}}{12}\right)\hbar^4+\left(-\frac{323}{8\,294\,400}+\frac{\gamma_{12}}{192}+\frac{\delta_{23}}{360}-\frac{7\,\delta_{30}}{4320}+\frac{\delta_{41}}{720}\right)\hbar^6+O[\hbar]^7\right.$$
$$\left.t[0]\right.$$
$$\left.h[0]\right.$$
$$\left.\frac{\hbar}{48}+\left(\frac{29}{5760}+\frac{\delta_{30}}{6}\right)\hbar^3+\left(\frac{1553}{58\,060\,800}+\frac{\gamma_{1}}{9}\right.\right.$$

No solutions to ek2==1!

Right cup top punctured.

```
ek3 = V * (C // dP[3] // dS[3]) // dm[3, 2, 2] // hη[2] // dS[1] // dm[2, 1, 0]
```

$$\left(1+\left(\frac{5\,c_0^2}{96}+\frac{1}{2}c_0^2\,\delta_{10}\right)\hbar^2+\left(\frac{143\,c_0^4}{30\,720}+\frac{5}{192}c_0^4\,\delta_{10}-\frac{1}{8}c_0^4\,\delta_{10}^2+\frac{1}{12}c_0^4\,\delta_{30}\right)\hbar^4+\left(\frac{35\,971\,c_0^6}{928\,972\,800}+\frac{1}{160}c_0^6\,\gamma_{12}+\frac{529\,c_0^6\,\delta_{1}}{921\,600}\right.\right.$$
$$\left.\left.t[0]\right.\right.$$

```
ek3 /. {c0 → 1, δ10 → -5 / 48}
```

$$\left(1+\left(\frac{3}{5120}+\frac{\delta_{30}}{12}\right)\hbar^4+\left(-\frac{239}{58\,060\,800}+\frac{13\,\gamma_{12}}{2880}+\frac{\delta_{23}}{360}+\frac{\delta_{30}}{1440}+\frac{\delta_{41}}{720}\right)\hbar^6+O[\hbar]^7\right.$$
$$\left.t[0]\right.$$
$$\left.-\frac{\hbar}{48}+\left(\frac{1}{640}+\frac{\delta_{30}}{6}\right)\hbar^3+\left(-\frac{2353}{58\,060\,80}\right.\right.$$

No solutions to ek3==1!